

Semileptonic Decays of B Meson Transition Into D -wave Charmed Meson Doublets

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We use QCD sum rules to estimate the leading-order universal form factors describing the semileptonic B decay into orbital excited D -wave charmed doublets, including the $(1^-, 2^-)$ states (D_1^*, D_2') and the $(2^-, 3^-)$ states (D_2, D_3^*) . The decay rates we predict are $\Gamma_{B \rightarrow D_1^* \ell \bar{\nu}} = \Gamma_{B \rightarrow D_2' \ell \bar{\nu}} = 2.4 \times 10^{-18} \text{GeV}$, $\Gamma_{B \rightarrow D_2 \ell \bar{\nu}} = 6.2 \times 10^{-17} \text{GeV}$, and $\Gamma_{B \rightarrow D_3^* \ell \bar{\nu}} = 8.6 \times 10^{-17} \text{GeV}$. The branching ratios are $\mathcal{B}(B \rightarrow D_1^* \ell \bar{\nu}) = \mathcal{B}(B \rightarrow D_2' \ell \bar{\nu}) = 6.0 \times 10^{-6}$, $\mathcal{B}(B \rightarrow D_2 \ell \bar{\nu}) = 1.5 \times 10^{-4}$, and $\mathcal{B}(B \rightarrow D_3^* \ell \bar{\nu}) = 2.1 \times 10^{-4}$, respectively.

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I. INTRODUCTION

Higher excitations than $D^{(*)}$ play an important role in the understanding of semileptonic B decays. Knowledge of these processes is important to reduce the uncertainties of the measurements on other semileptonic B decays, and thus the determination of the Cabibbo-Kobayashi-Maskawa matrix elements, such as $|V_{cb}|$. Theoretically, the semileptonic decay processes are described by some form factors. The challenge for theory is the calculation of these decay form factors. Fortunately, the heavy quark effective theory (HQET) [1], with an expansion in terms of $1/m_Q$ for hadrons containing a single heavy quark, provides a systematic method for investigating such processes. In HQET the approximate symmetries allow one to organize the spectrum of heavy mesons according to parity P and total angular momentum s_l of the light degree of freedom. Coupling the spin of the light degrees of freedom s_l with the spin of a heavy quark $s_Q = 1/2$ yields a doublet of meson states with a total spin $s = s_l \pm 1/2$. For charmed mesons, the lowest lying states $(0^-, 1^-)$ doublet (D, D^*) are S -wave states with the spin of light degrees $s_l = 1/2$. The P -wave excitation corresponds to two series of states, one is the $s_l = 1/2$ series, the $(0^+, 1^+)$ doublet (D_0^*, D_1') ; the other is the $s_l = 3/2$ series, the $(1^+, 2^+)$ doublet (D_1, D_2^*) . For D -wave states, those are $(1^-, 2^-)$ and $(2^-, 3^-)$ doublets $((D_1^*, D_2')$ and (D_2, D_3^*)), corresponding to the spin of light degrees of freedom $s_l = 3/2$ and $s_l = 5/2$. The early study of the heavy-light mesons can be found in Ref. [2]. The S -wave and P -wave charmed states have been observed so far. The

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properties of these states have been extensively studied using different approaches during the past few years, including masses [3, 4], decay constants [5, 6, 7], and decay widths [8, 9, 10, 11]. For the D -wave charmed mesons, their properties were investigated with the potential model [10] and QCD sum rules [12].

Semileptonic B decay into an excited heavy meson has been observed in experiments [13, 14]. Recently, BABAR has measured semileptonic B decays into orbitally excited charmed mesons $D_1(2420)$ and $D_2^*(2460)$ [15]. They also reported two new D_s states $D_{sJ}(2860)$ and $D_{sJ}(2690)$ in the DK channel, which may fit in the D -wave charm-strange doublets [16]. A similar state $D_{sJ}(2715)$ has also been observed by Belle [17]. It is expected that the nonstrange D -wave charmed mesons will be found, and the measurements of the semileptonic B decays into these states become available in the near future. To this end we study the predictions of HQET for semileptonic B decays to D -wave charmed mesons.

The semileptonic decay rate of a B meson transition into an charmed meson is determined by the corresponding matrix elements of the weak axial-vector and vector currents. In the heavy quark limit these elements are described, respectively, by one universal Isgur-Wise function at the leading order of heavy quark expansion [18]. The universal Isgur-Wise function is a nonperturbative parameter. It must be calculated in some nonperturbative approaches. The main theoretical approaches are QCD sum rules [19], constituent quark models, and lattice QCD. The investigations of semileptonic B decays into charmed mesons can be found in Refs. [5, 18, 20, 21, 22] with different methods. In this work, we estimate the leading-order Isgur-Wise functions describing the decays $B \rightarrow (D_1^*, D_2')\ell\bar{\nu}$ and $B \rightarrow (D_2, D_3^*)\ell\bar{\nu}$ and give a prediction for the widths of the decays.

The remainder of this paper is organized as follows. In Sec. II we present the formulas of weak current matrix elements and decay rates. In Sec. III we give the relevant sum rules for two-point correlators, and then deduce the three-point sum rules for the Isgur-Wise functions. Section IV is devoted to numerical results and discussions.

II. ANALYTIC FORMULATIONS FOR SEMILEPTONIC DECAY

AMPLITUDES $B \rightarrow (D_1^*, D_2')\ell\bar{\nu}$ AND $B \rightarrow (D_2, D_3^*)\ell\bar{\nu}$

The heavy-light meson doublets can be expressed conveniently by effective operators [23]. For the ground doublet, the operator is

$$H_a = \frac{1 + \not{v}}{2} [D_\mu^* \gamma^\mu - D \gamma_5]. \quad (1)$$

The effective operators describing the meson doublets $D(1^-, 2^-)$ and $D(2^-, 3^-)$ are given by

$$X^\mu = \frac{1 + \not{v}}{2} [D_2^{\prime\mu\nu} \gamma_5 \gamma_\nu - D_{1\nu}^* \sqrt{\frac{3}{2}} (g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu + v^\mu))], \quad (2)$$

and

$$H^{\mu\nu} = \frac{1+\not{v}}{2} [D_3^{*\mu\nu\sigma} \gamma_\sigma - \sqrt{\frac{3}{5}} \gamma_5 D_2^{\alpha\beta} (g_\alpha^\mu g_\beta^\nu - \frac{\gamma_\alpha}{5} g_\beta^\nu (\gamma^\mu - v^\mu) - \frac{\gamma_\beta}{5} g_\alpha^\mu (\gamma^\nu - v^\nu))] . \quad (3)$$

In these operators, D_μ^* , D , $D_2^{\prime\mu\nu}$, $D_{1\nu}^*$, $D_3^{*\mu\nu\sigma}$, and $D_2^{\alpha\beta}$ separately represent annihilation operators of the $Q\bar{q}$ mesons with appropriate quantum numbers and $\not{v} = v \cdot \gamma$, v is the heavy meson velocity. The theoretical description of semileptonic decays involves the matrix elements of vector and axial-vector currents ($V^\mu = \bar{c}\gamma^\mu b$ and $A^\mu = \bar{c}\gamma^\mu \gamma_5 b$) between B mesons and excited D mesons. For the processes $B \rightarrow (D_1^*, D_2') \ell \bar{\nu}$ and $B \rightarrow (D_2, D_3^*) \ell \bar{\nu}$, these matrix elements can be parametrized through applying the trace formalism as follows [23]:

$$\begin{aligned} \langle D_1^*(v', \varepsilon) | (V - A)^\mu | B(v) \rangle &= \sqrt{\frac{3}{2}} \sqrt{m_B m_{D_1^*}} \tau_1(y) [\varepsilon^* \cdot v (v^\mu - \frac{y+2}{3} v'^\mu) \\ &\quad - i \frac{1-y}{3} \epsilon^{\mu\alpha\beta\sigma} \varepsilon_\alpha^* v'_\beta v_\sigma], \end{aligned} \quad (4)$$

$$\langle D_2'(v', \varepsilon) | (V - A)^\mu | B(v) \rangle = \sqrt{m_B m_{D_2'}} \tau_1(y) \varepsilon_{\rho\nu}^* v^\rho [g^{\mu\nu} (y-1) - v^\nu v'^\mu + i \epsilon^{\alpha\beta\gamma\mu} v'_\alpha v_\beta], \quad (5)$$

and

$$\begin{aligned} \langle D_2(v', \varepsilon) | (V - A)^\mu | B(v) \rangle &= \sqrt{\frac{5}{3}} \sqrt{m_B m_{D_2}} \tau_2(y) \varepsilon_{\alpha\beta}^* v^\alpha [\frac{2(1-y^2)}{5} g^{\mu\beta} - v^\beta v'^\mu + \frac{2y-3}{5} v'^\beta v'^\mu \\ &\quad + i \frac{2(1+y)}{5} \epsilon^{\mu\lambda\beta\rho} v_\lambda v'_\rho], \end{aligned} \quad (6)$$

$$\langle D_3^*(v', \varepsilon) | (V - A)^\mu | B(v) \rangle = \sqrt{m_B m_{D_3^*}} \tau_2(y) \varepsilon_{\alpha\beta\lambda}^* v^\alpha v^\beta [g^{\mu\lambda} (1+y) - v^\lambda v'^\mu + i \epsilon^{\mu\lambda\rho\tau} v_\rho v'_\tau], \quad (7)$$

where $(V - A)^\mu = \bar{c}\gamma^\mu(1 - \gamma_5)b$ is the weak current, $y = v \cdot v'$ and $\tau_1(y)$, $\tau_2(y)$ are the universal form factors, and ε_α^* , $\varepsilon_{\alpha\beta}^*$, $\varepsilon_{\alpha\beta\lambda}^*$ are the polarization tensors of these mesons. The differential decay rates are calculated by making use of the formulas (4) to (7) given above:

$$\frac{d\Gamma}{dy}(B \rightarrow D_1^* \ell \bar{\nu}) = \frac{G_F^2 V_{cb}^2 m_B^2 m_{D_1^*}^3}{72\pi^3} (\tau_1(y))^2 (y-1)^{\frac{5}{2}} (y+1)^{\frac{3}{2}} [(1+r_1^2)(2y+1) - 2r_1(y^2+y+1)], \quad (8)$$

$$\frac{d\Gamma}{dy}(B \rightarrow D_2' \ell \bar{\nu}) = \frac{G_F^2 V_{cb}^2 m_B^2 m_{D_2'}^3}{72\pi^3} (\tau_1(y))^2 (y-1)^{\frac{5}{2}} (y+1)^{\frac{3}{2}} [(1+r_2^2)(4y-1) - 2r_2(3y^2-y+1)], \quad (9)$$

$$\frac{d\Gamma}{dy}(B \rightarrow D_2 \ell \bar{\nu}) = \frac{G_F^2 V_{cb}^2 m_B^2 m_{D_2}^3}{360\pi^3} (\tau_2(y))^2 (y-1)^{\frac{5}{2}} (y+1)^{\frac{7}{2}} [(1+r_3^2)(7y-3) - 2r_3(4y^2-3y+3)], \quad (10)$$

$$\frac{d\Gamma}{dy}(B \rightarrow D_3^* \ell \bar{\nu}) = \frac{G_F^2 V_{cb}^2 m_B^2 m_{D_3^*}^3}{360\pi^3} (\tau_2(y))^2 (y-1)^{\frac{5}{2}} (y+1)^{\frac{7}{2}} [(1+r_4^2)(11y+3) - 2r_4(8y^2+3y+3)], \quad (11)$$

with $r_i = \frac{m_{D_i}}{m_B}$ ($D_i = D_1^*, D_2', D_2, D_3^*$ for $i = 1, 2, 3, 4$). In the equations above, we have presented the decay rates of B semileptonic decay processes $B \rightarrow (D_1^*, D_2') \ell \bar{\nu}$ and $B \rightarrow (D_2, D_3^*) \ell \bar{\nu}$ in terms of the universal form factors $\tau_1(y)$ and $\tau_2(y)$, respectively. The only unknown factors in these equations are $\tau_1(y)$ and $\tau_2(y)$, which need to be determined by nonperturbative methods.

III. SUM RULES FOR ISGUR-WISE FUNCTIONS

In the calculation of Isgur-Wise functions in HQET by means of QCD sum rule, the interpolating currents are potentially important. In Ref. [4], two series of interpolating currents with nice properties were proposed:

$$J_{j,P,i}^{\dagger\alpha_1\cdots\alpha_j} = \bar{h}_v(x) \Gamma_{j,P,i}^{\{\alpha_1\cdots\alpha_j\}}(D_{x_t}) q(x) \quad (12)$$

or

$$J_{j,P,i}'^{\dagger\alpha_1\cdots\alpha_j} = \bar{h}_v(x) \Gamma_{j,P,i}^{\{\alpha_1\cdots\alpha_j\}}(D_{x_t})(-i) \not{D}_{x_t} q(x) \quad (13)$$

where $i = 1, 2$ corresponding to two series of doublets of the spin-parity $[j^{(-1)^{j+1}}, (j+1)^{(-1)^{j+1}}]$ and $[j^{(-1)^j}, (j+1)^{(-1)^j}]$, respectively. $D_{t\mu} = D_\mu - v_\mu(v \cdot D)$ is the transverse component of the covariant derivative with respect to the velocity of the meson and

$$\Gamma^{\{\alpha_1\cdots\alpha_j\}}(D_{x_t}) = \text{symmetrize}\{\Gamma^{\alpha_1\cdots\alpha_j}(D_{x_t}) - \frac{1}{3}g_t^{\alpha_1\alpha_2}g_{\alpha'_1\alpha'_2}^t\Gamma^{\alpha'_1\alpha'_2\alpha_3\cdots\alpha_j}\} \quad (14)$$

with the transverse metric $g_t^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta$. For the doublets of spin-parity $[j^{(-1)^{j+1}}, (j+1)^{(-1)^{j+1}}]$ and $[j^{(-1)^j}, (j+1)^{(-1)^j}]$, the expressions for $\Gamma^{\alpha_1\cdots\alpha_j}(D_{x_t})$ have been explicitly given in [4] as

$$\Gamma(D_{x_t}) = \begin{cases} \sqrt{\frac{2j+1}{2j+2}}\gamma^5(-i)^j D_{x_t}^{\alpha_2} \cdots D_{x_t}^{\alpha_j} (D_{x_t}^{\alpha_1} - \frac{j}{2j+1}\gamma_t^{\alpha_1} \not{D}_{x_t}), & \text{for } j^{(-1)^{j+1}} \\ \frac{1}{\sqrt{2}}\gamma_t^{\alpha_1}(-i)^j D_{x_t}^{\alpha_2} \cdots D_{x_t}^{\alpha_j}, & \text{for } (j+1)^{(-1)^{j+1}} \end{cases}$$

$$\Gamma(D_{x_t}) = \begin{cases} \frac{1}{\sqrt{2}}\gamma^5(-i)^j \gamma_t^{\alpha_1} D_{x_t}^{\alpha_2} \cdots D_{x_t}^{\alpha_{j+1}}, & \text{for } (j+1)^{(-1)^j} \\ \sqrt{\frac{2j+1}{2j+2}}(-i)^j D_{x_t}^{\alpha_2} \cdots D_{x_t}^{\alpha_j} (D_{x_t}^{\alpha_1} - \frac{j}{2j+1}\gamma_t^{\alpha_1} \not{D}_{x_t}), & \text{for } j^{(-1)^j} \end{cases}$$

where $\gamma_{t\mu} = \gamma_\mu - \not{v}v_\mu$ is the transverse component of γ_μ with respect to the heavy quark velocity.

For the D -wave meson doublets with $s_l = \frac{3}{2}^-$ and $s_l = \frac{5}{2}^-$, where $j = 1$ and $j = 2$, the currents are given by the following expressions:

$$J_{1,-,3/2}^{\dagger\alpha} = -i\sqrt{\frac{3}{4}}\bar{h}_v(D_t^\alpha - \frac{1}{3}\gamma_t^\alpha \not{D}_t)q, \quad (15)$$

$$J_{2,-,3/2}^{\dagger\alpha\beta\lambda} = -i\frac{1}{\sqrt{2}}T^{\alpha\beta,\mu\nu}\bar{h}_v\gamma_5\gamma_{t\mu}D_{t\nu}q, \quad (16)$$

and

$$J_{2,-,5/2}^{\dagger\alpha\beta} = -\sqrt{\frac{5}{6}}T^{\alpha\beta,\mu\nu}\bar{h}_v\gamma_5(D_{t\mu}D_{t\nu} - \frac{2}{5}D_{t\mu}\gamma_{t\nu}\not{D}_t)q, \quad (17)$$

$$J_{3,-,5/2}^{\dagger\alpha\beta\lambda} = -\frac{1}{\sqrt{2}}T^{\alpha\beta\lambda,\mu\nu\sigma}\bar{h}_v\gamma_{t\mu}D_{t\nu}D_{t\sigma}q, \quad (18)$$

which correspond to Eq. (12), and corresponding to Eq. (13) are

$$J_{1,-,3/2}^{\dagger\alpha} = -\sqrt{\frac{3}{4}}\bar{h}_v(D_t^\alpha - \frac{1}{3}\gamma_t^\alpha \not{D}_t)\not{D}_t q, \quad (19)$$

$$J_{2,-,3/2}^{\dagger\alpha\beta\lambda} = -\frac{1}{\sqrt{2}}T^{\alpha\beta,\mu\nu}\bar{h}_v\gamma_5\gamma_{t\mu}D_{t\nu}P_tq, \quad (20)$$

and

$$J_{2,-,5/2}^{\dagger\alpha\beta} = -\sqrt{\frac{5}{6}}T^{\alpha\beta,\mu\nu}\bar{h}_v\gamma_5(D_{t\mu}D_{t\nu} - \frac{2}{5}D_{t\mu}\gamma_{t\nu}P_t)(-i)P_tq, \quad (21)$$

$$J_{3,-,5/2}^{\dagger\alpha\beta\lambda} = -\frac{1}{\sqrt{2}}T^{\alpha\beta\lambda,\mu\nu\sigma}\bar{h}_v\gamma_{t\mu}D_{t\nu}D_{t\sigma}(-i)P_tq, \quad (22)$$

where h_v is the generic velocity-dependent heavy quark effective field in HQET and q denotes the light quark field. The tensors $T^{\alpha\beta,\mu\nu}$ and $T^{\alpha\beta\lambda,\mu\nu\sigma}$ are used to symmetrize indices and are given by [4]

$$T^{\alpha\beta,\mu\nu} = \frac{1}{2}(g_t^{\alpha\mu}g_t^{\beta\nu} + g_t^{\alpha\nu}g_t^{\beta\mu}) - \frac{1}{3}g_t^{\alpha\beta}g_t^{\mu\nu}, \quad (23)$$

$$\begin{aligned} T^{\alpha\beta\lambda,\mu\nu\sigma} = & \frac{1}{6}(g_t^{\alpha\mu}g_t^{\beta\nu}g_t^{\lambda\sigma} + g_t^{\alpha\mu}g_t^{\beta\sigma}g_t^{\lambda\nu} + g_t^{\alpha\nu}g_t^{\beta\mu}g_t^{\lambda\sigma} + g_t^{\alpha\nu}g_t^{\beta\sigma}g_t^{\lambda\mu} + g_t^{\alpha\sigma}g_t^{\beta\nu}g_t^{\lambda\mu} + g_t^{\alpha\sigma}g_t^{\beta\mu}g_t^{\lambda\nu}) \\ & - \frac{1}{15}(g_t^{\alpha\beta}g_t^{\mu\nu}g_t^{\lambda\sigma} + g_t^{\alpha\beta}g_t^{\mu\sigma}g_t^{\lambda\nu} + g_t^{\alpha\beta}g_t^{\nu\sigma}g_t^{\lambda\mu} + g_t^{\alpha\lambda}g_t^{\mu\nu}g_t^{\beta\sigma} + g_t^{\alpha\lambda}g_t^{\mu\sigma}g_t^{\beta\nu} \\ & + g_t^{\alpha\lambda}g_t^{\nu\sigma}g_t^{\beta\mu} + g_t^{\beta\lambda}g_t^{\mu\nu}g_t^{\alpha\sigma} + g_t^{\beta\lambda}g_t^{\mu\sigma}g_t^{\alpha\nu} + g_t^{\beta\lambda}g_t^{\nu\sigma}g_t^{\alpha\mu}). \end{aligned} \quad (24)$$

Usually the currents with derivatives of the lowest order (12) are used in the QCD sum rule approach. However, currents with derivatives of one order higher (13) are also used in some conditions because in the nonrelativistic quark model there is a corresponding relation between the orbital angular momenta and the orders of derivatives in the space wave functions. As for the orbital D-wave mesons, which corresponding to derivatives of order two, it is reasonable to use the currents (17), (18), (19) and (20).

These currents have nice properties, they have nonvanishing projection only to the corresponding states of the HQET in the $m_Q \rightarrow \infty$ limit, without mixing with states of the same quantum number but different s_l . Thus we can define one-particle-current couplings as follows:

$$J^P = 1^- : \langle D_1^*(v, \varepsilon) | J^\alpha | 0 \rangle = f_1 \sqrt{m_{D_1^*}} \varepsilon^{*\alpha}, \quad (25)$$

$$J^P = 2^- : \langle D_2'(v, \varepsilon) | J^{\alpha\beta} | 0 \rangle = f_2' \sqrt{m_{D_2'}} \varepsilon^{*\alpha\beta}, \quad (26)$$

$$J^P = 2^- : \langle D_2(v, \varepsilon) | J^{\alpha\beta} | 0 \rangle = f_2 \sqrt{m_{D_2}} \varepsilon^{*\alpha\beta}, \quad (27)$$

$$J^P = 3^- : \langle D_3^*(v, \varepsilon) | J^{\alpha\beta\lambda} | 0 \rangle = f_3 \sqrt{m_{D_3^*}} \varepsilon^{*\alpha\beta\lambda}. \quad (28)$$

The couplings f_i are low-energy parameters which are determined by the dynamics of the light degree of freedom. Since the pairs (f_1, f_2') and (f_2, f_3) are related by the spin symmetry, we will consider f_1 and f_2 hereafter. The decay constants f_i can be estimated from two-point sum rules, therefore we list the sum rules after the Borel transformation.

For the ground-state heavy mesons, the sum rule for the correlator of two heavy-light currents is well known. It is [20]

$$f_{-, \frac{1}{2}}^2 e^{-2\bar{\Lambda}_{-, \frac{1}{2}}/T} = \frac{3}{16\pi^2} \int_0^{\omega_{c0}} \omega^2 e^{-\omega/T} d\omega - \frac{1}{2} \langle \bar{q}q \rangle (1 - \frac{m_0^2}{4T^2}). \quad (29)$$

For the $s_l^P = \frac{3}{2}^-$ doublet, when the currents (19) and (20) are used, the corresponding sum rule is :

$$f_{-, \frac{3}{2}}^2 e^{-2\bar{\Lambda}_{-, \frac{3}{2}}/T} = \frac{1}{2^8 \pi^2} \int_0^{\omega_{c1}} \omega^6 e^{-\omega/T} d\omega - \frac{5}{3 \times 2^8} \int_0^{\omega_{c1}} \omega^2 e^{-\omega/T} d\omega \langle \frac{\alpha_s}{\pi} GG \rangle. \quad (30)$$

For the $s_l^P = \frac{5}{2}^-$ doublet, when the currents (17) and (18) are used, the corresponding sum rule is :

$$f_{-, \frac{5}{2}}^2 e^{-2\bar{\Lambda}_{-, \frac{5}{2}}/T} = \frac{1}{5 \times 2^7 \pi^2} \int_0^{\omega_{c2}} \omega^6 e^{-\omega/T} d\omega - \frac{5}{3 \times 2^6} \int_0^{\omega_{c2}} \omega^2 e^{-\omega/T} d\omega \langle \frac{\alpha_s}{\pi} GG \rangle. \quad (31)$$

As we have just mentioned, for the amplitudes of the semileptonic decays into excited states in the infinite mass limit, the only unknown quantities in (8), (9), (10) and (11) are the universal functions $\tau_1(y)$ and $\tau_2(y)$. In Ref. [24] the form factors $\tau_1(y)$ and $\tau_2(y)$ were estimated through QCD sum rule by using currents with derivatives of lower order, (15) to (18). Considering that the corresponding relation between the orbital angular momentum and the order of the derivative mentioned above, we use the currents (19) and (20) instead of (15) and (16) for the (D_1^*, D_2') doublet. As for the (D_2, D_3^*) doublet, we also use the currents (17) and (18).

In order to calculate this two form factors by QCD sum rules, we study the analytic properties of three-point correlators:

$$i^2 \int d^4x d^4z e^{i(k' \cdot x - k \cdot z)} \langle 0 | T [J_{1,-}^\alpha(x) J_{V,A}^{\mu(v,v')}(0) J_{0,-}^\dagger(z) | 0 \rangle = \Gamma(\omega, \omega', y) \mathcal{L}_{V,A}^{\mu\alpha}, \quad (32)$$

$$i^2 \int d^4x d^4z e^{i(k' \cdot x - k \cdot z)} \langle 0 | T [J_{2,-}^{\alpha\beta}(x) J_{V,A}^{\mu(v,v')}(0) J_{0,-}^\dagger(z) | 0 \rangle = \Gamma'(\omega, \omega', y) \mathcal{L}_{V,A}^{\mu\alpha\beta}, \quad (33)$$

where $J_V^{\mu(v,v')} = h(v') \gamma^\mu h(v)$ and $J_A^{\mu(v,v')} = h(v') \gamma^\mu \gamma_5 h(v)$. The variables $k(= P - m_b v)$ and $k'(= P' - m_c v')$ denote residual “off-shell” momenta of the initial and final meson states, respectively. For heavy quarks in bound states they are typically of order Λ_{QCD} and remain finite in the heavy quark limit. $\Gamma(\omega, \omega', y)$ and $\Gamma'(\omega, \omega', y)$ are analytic functions in the “off-shell” energies $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$ with discontinuities for positive values of these variables. They also depend on the velocity transfer $y = v \cdot v'$, which is fixed in a physical region. $\mathcal{L}_{V,A}$ are Lorentz structures.

Following the standard QCD sum rule procedure, the calculations of $\Gamma(\omega, \omega', y)$ and $\Gamma'(\omega, \omega', y)$ are straightforward. First, we saturate Eqs.(32) and (33) with physical intermediate states in HQET and find that the hadronic representations of the correlators as follows:

$$\Gamma_{hadron}(\omega, \omega', y) = \frac{f_{-, \frac{1}{2}} f_{-, j_l} \tau_i(y)}{(2\bar{\Lambda}_{-, \frac{1}{2}} - \omega - i\varepsilon)(2\bar{\Lambda}_{-, j_l} - \omega' - i\varepsilon)} + \text{higher resonances}, \quad (34)$$

where f_{-,j_l} are the decay constants defined in Eqs.(25) and (27), $\bar{\Lambda}_{-,j_l} = m_{-,j_l} - m_Q$. Second, the functions can be approximated by a perturbative calculation supplemented by nonperturbative power corrections proportional to the vacuum condensates which are treated as phenomenological parameters. The perturbative contribution can be represented by a double dispersion integral in ν and ν' plus possible subtraction terms. So the theoretical expression for the correlator has the form

$$\Gamma_{theo}(\omega, \omega', y) \simeq \int d\nu d\nu' \frac{\rho^{pert}(\nu, \nu', y)}{(\nu - \omega - i\varepsilon)(\nu' - \omega' - i\varepsilon)} + \text{subtractions} + \Gamma^{cond}(\omega, \omega', y). \quad (35)$$

The perturbative part of the spectral density can be calculated straightforward. Confining us to the leading order of perturbation, the perturbative spectral densities of the two sum rules for $\tau_1(y)$ and $\tau_2(y)$ are

$$\begin{aligned} \rho_{pert}(\nu, \nu', y) &= \frac{3}{2^8 \pi^2} \frac{1}{(y+1)^{\frac{3}{2}} (y-1)^{\frac{5}{2}}} \nu' [(3\nu^2 - (1+2y)(2\nu\nu' - \nu'^2)] \\ &\quad \times \Theta(\nu) \Theta(\nu') \Theta(2y\nu\nu' - \nu^2 - \nu'^2), \end{aligned} \quad (36)$$

and

$$\begin{aligned} \rho_{pert}(\nu, \nu', y) &= \frac{3}{2^8 \pi^2} \frac{1}{(y+1)^{\frac{1}{2}} (y-1)^{\frac{5}{2}}} [(5\nu - 12y\nu' + 3\nu')\nu^2 + (3\nu + \nu')(2y^2 - 2y + 1)\nu'^2] \\ &\quad \times \Theta(\nu) \Theta(\nu') \Theta(2y\nu\nu' - \nu^2 - \nu'^2). \end{aligned} \quad (37)$$

Following the arguments in Refs. [5, 25], the perturbative and the hadronic spectral densities cannot be locally dual to each other, the necessary way to restore duality is to integrate the spectral densities over the “off-diagonal” variable $\nu_- = \nu - \nu'$, keeping the “diagonal” variable $\nu_+ = \frac{\nu + \nu'}{2}$ fixed. It is in ν_+ that the quark-hadron duality is assumed for the integrated spectral densities. The integration region can be expressed in terms of the variables ν_- and ν_+ and we choose the triangular region defined by the bounds: $0 \leq \nu_+ \leq \omega_c$, $-2\sqrt{\frac{y-1}{y+1}}\nu_+ \leq \nu_- \leq 2\sqrt{\frac{y-1}{y+1}}\nu_+$. As discussed in Refs. [5, 25], the upper limit ω_c for ν_+ in the region $\frac{1}{2}[(y+1) - \sqrt{y^2 - 1}]\omega_{c0} \leq \omega_c \leq \frac{1}{2}(\omega_{c0} + \omega_{c2})$ is reasonable. A double Borel transformation in ω and ω' is performed on both sides of the sum rules, in which for simplicity we take the Borel parameters equal [5, 20, 24]: $T_1 = T_2 = 2T$. In the calculation, we have considered the operators of dimension $D \leq 5$ in OPE. After adding the nonperturbative parts, we obtain the sum rules for τ_1 and τ_2 as follows:

$$\begin{aligned} \tau_1(y) f_{-,1/2} f_{-,3/2} e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{-,3/2})/T} &= \frac{1}{2^4 \pi^2} \frac{1}{(1+y)^3} \int_0^{\omega_c'} d\nu_+ e^{-\frac{\nu_+}{T}} \nu_+^4 \\ &\quad - \frac{T}{3 \times 2^5} \frac{2y+3}{(y+1)^2} \langle \frac{\alpha_s}{\pi} GG \rangle, \end{aligned} \quad (38)$$

$$\begin{aligned} \tau_2(y) f_{-,1/2} f_{-,5/2} e^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{-,5/2})/T} &= \frac{3}{8\pi^2} \frac{1}{(1+y)^4} \int_0^{\omega_c} d\nu_+ e^{-\frac{\nu_+}{T}} \nu_+^4 \\ &\quad - \frac{T}{3 \times 2^4} \frac{1}{(y+1)^3} \langle \frac{\alpha_s}{\pi} GG \rangle. \end{aligned} \quad (39)$$

We also derive the sum rule for τ_2 by using the currents (21) and (22), which appears to be

$$\begin{aligned} \tau_2(y)f_{-,1/2}f_{-,5/2}e^{-(\bar{\Lambda}_{-,1/2}+\bar{\Lambda}_{-,5/2})/T} &= \frac{21}{5 \times 2^4 \pi^2} \frac{1}{(1+y)^4} \int_0^{\omega_c} d\nu_+ e^{-\frac{\nu_+}{T}} \nu_+^5 \\ &+ \frac{T^2}{3 \times 2^4} \frac{4y-25}{(y+1)^3} \langle \frac{\alpha_s}{\pi} GG \rangle. \end{aligned} \quad (40)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

We now evaluate the sum rules numerically. For the QCD parameters entering the theoretical expressions, we take the standard values: $\langle \bar{q}q \rangle = -(0.24)^3 \text{GeV}^3$, $\langle \alpha_s GG \rangle = 0.04 \text{GeV}^4$, and $m_0^2 = 0.8 \text{GeV}^2$. In the numerical calculations, we take 2.83GeV [2, 10] for the mass of the $s_l = 5/2$ doublet and 2.78GeV for the $s_l = 3/2$ doublet. For mass of initial B meson, we use $m_B = 5.279 \text{GeV}$ [26].

In order to obtain information of $\tau_1(y)$ and $\tau_2(y)$ with less systematic uncertainties in the calculation, we divide the three-point sum rules by the square roots of relevant two-point sum rules, as many authors did [5, 20, 24], to reduce the number of input parameters and improve stabilities. Then we obtain expressions for the $\tau_1(y)$ and $\tau_2(y)$ as functions of the Borel parameter T and the continuum thresholds. Imposing usual criteria for the upper and lower bounds of the Borel parameter, we found they have a common sum rule “window”: $0.7 \text{GeV} < T < 1.5 \text{GeV}$, which overlaps with those of two-point sum rules (29), (30) and (31) (see Fig. 1). Notice that the Borel parameter in the sum rules for three-point correlators is twice the Borel parameter in the sum rules for the two-point correlators. In the evaluation we have taken $2.0 \text{GeV} < \omega_{c0} < 2.4 \text{GeV}$ [5, 20], $2.8 \text{GeV} < \omega_{c1} < 3.2 \text{GeV}$, and $3.2 \text{GeV} < \omega_{c2} < 3.6 \text{GeV}$. The regions of these continuum thresholds are fixed by analyzing the corresponding two-point sum rules. According to the discussion in Sec. III, we can fix ω'_c and ω_c in the regions $2.3 \text{GeV} < \omega'_c < 2.6 \text{GeV}$ and $2.5 \text{GeV} < \omega_c < 2.7 \text{GeV}$. The results are showed in Fig. 2. The resulting curves for $\tau_1(y)$ and $\tau_2(y)$ can be parametrized by the linear approximation

$$\tau_1(y) = \tau_1(1)[1 - \rho_{\tau_1}^2(y-1)], \quad \tau_1(1) = 0.14 \pm 0.03, \quad \rho_{\tau_1}^2 = 0.13 \pm 0.02; \quad (41)$$

$$\tau_2(y) = \tau_2(1)[1 - \rho_{\tau_2}^2(y-1)], \quad \tau_2(1) = 0.57 \pm 0.09, \quad \rho_{\tau_2}^2 = 0.78 \pm 0.13. \quad (42)$$

The errors mainly come from the uncertainty due to ω_c 's and T . It is difficult to estimate these systematic errors which are brought in by the quark-hadron duality. The maximal values of y are $y_{max}^{D_1^*} = y_{max}^{D_2'} = (1 + r_{1,2}^2)/2r_{1,2} \approx 1.213$ and $y_{max}^{D_2} = y_{max}^{D_3^*} = (1 + r_{3,4}^2)/2r_{3,4} \approx 1.201$. By using the parameters $V_{cb} = 0.04$, $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$, we get the semileptonic decay rates of $B \rightarrow (D_1^*, D_2')\ell\bar{\nu}$ and $B \rightarrow (D_2, D_3^*)\ell\bar{\nu}$. Consider that $\tau_B = 1.638 \text{ps}$ [26], we get the branching ratios, respectively. All these results are listed in Table I.

Because of the large background from $B \rightarrow D^{(*)}\ell\bar{\nu}$ decays, there is no experimental data available so far. As we can see from Table I, the rates of semileptonic B decay into the

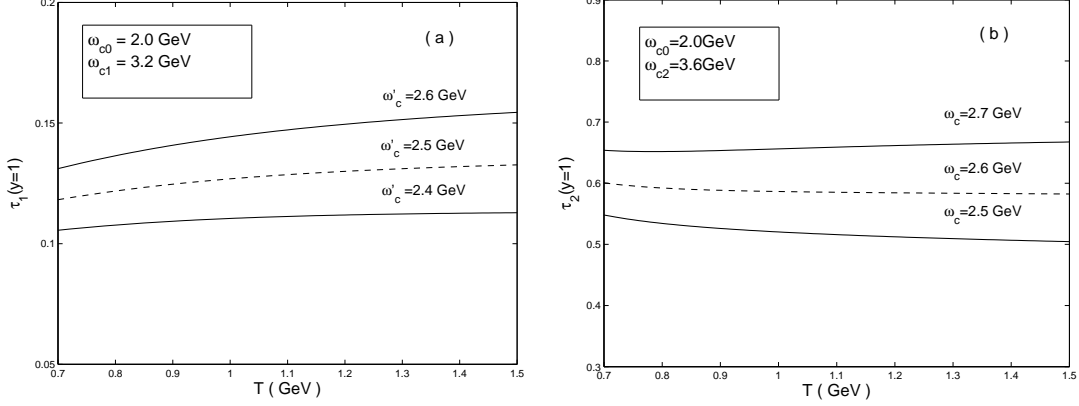


FIG. 1: Dependence of $\tau_1(y)$ and $\tau_2(y)$ on Borel parameter T at $y = 1$.

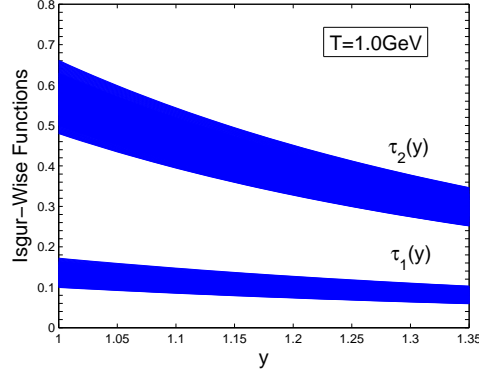


FIG. 2: Prediction for the Isgur-Wise functions $\tau_1(y)$ and $\tau_2(y)$.

$s_l^P = \frac{3}{2}^-$ doublet are tiny and our results are larger than those predicted by Ref. [24] in the B to $s_l^P = \frac{5}{2}^-$ charmed doublet channels. The difference comes because the way in which we choose the parameters is different from theirs. They chose the parameters according to other theoretical approaches. In contrast, we choose the parameters following the way of Ref. [5]. In addition, we also estimate the universal form factor $\tau_2(y)$ with the sum rule (40) and we get almost the same result as (42). When trying to estimate the $\tau_1(y)$ by using the currents (15) and (16), we find that after the quark-hadron duality are assumed the integral over the perturbative spectral density becomes zero. As for the P -wave and the F -wave mesons, similar results can be obtained after the calculations above have been carefully repeated.

The semileptonic and leptonic B decay rate is about 10.9% of the total B decay rate, in which the S -wave charmed mesons D and D^* contribute about 8.65% [26] and the P -wave charmed mesons contribute about 0.9% [20]. Our results then suggest that the D -wave charmed mesons contribute about 0.04% of the total B decay rate. Sum up the branching ratios of these semileptonic B decay processes, the eight lightest charmed mesons

TABLE I: Predictions for the decay widths and branching ratios

Decay mode	Decay width Γ (GeV)	Branching ratio	Branching ratio of Ref.[24]
$B \rightarrow D_1^* \ell \bar{\nu}$	2.4×10^{-18}	6.0×10^{-6}	
$B \rightarrow D_2' \ell \bar{\nu}$	2.4×10^{-18}	6.0×10^{-6}	
$B \rightarrow D_2 \ell \bar{\nu}$	6.2×10^{-17}	1.5×10^{-4}	1×10^{-5}
$B \rightarrow D_3^* \ell \bar{\nu}$	8.6×10^{-17}	2.1×10^{-4}	1×10^{-5}

contribute about 9.59% of the B decay rate. Therefore, semileptonic decays into higher excited states and nonresonant multibody channels should be about 1.31% of the B decay rate. Whatsoever, our result is just a leading-order estimate of the contribution of the D -wave charmed mesons channels to the semileptonic B decay.

In summary, we estimate the leading-order universal form factors describing the B meson of ground-state transition into orbital excited D -wave charmed resonances, the $(1^-, 2^-)$ states (D_1^*, D_2') , which belong to the $s_l^P = \frac{3}{2}^-$ heavy quark doublet and the $(2^-, 3^-)$ states (D_2, D_3^*) , which belong to the $s_l^P = \frac{5}{2}^-$ heavy quark doublet, by use of QCD sum rules within the framework of HQET. The semileptonic decay widths as well as the branching ratios we get are shown in Table I. The predictions are larger than those predicted by Ref. [24]. This needs future experiments for clarification. We also prove that when $s_l^P = \frac{5}{2}^-$ the interpolating currents (12) and (13) proposed in Ref. [4] are really equivalent. It is worth noting that in the estimate of the semileptonic B decay form factors when the currents (12) with quantum numbers of light degree of freedom $s_l^P = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+$ are used for the excited charmed mesons, we find the perturbative contributions vanish after the quark-hadron duality are assumed. In this case we should use the currents (13) which contain derivatives of one order higher.

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